# Frequency conversion of continuous variable quantum states

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The frequency conversion of cw continuous variable quantum states via second-order nonlinear optical processes is theoretically analyzed. It is shown that wideband frequency conversion of cw continuous variable quantum states is feasible by sum-frequency generation. For difference-frequency generation, besides as an optimal-phase-conjugation frequency-conversion process, any one quadrature of the quantum state can be frequency converted through it. Particularly, we analyze the influence of pump field fluctuations on the fidelity of the frequency-conversion process for a coherent state input. © 2008 Optical Society of America

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## 1. INTRODUCTION

In quantum information processing and communication, quantum interfaces capable of transferring quantum states from one type of carrier to another are potentially useful. These include the ability to shift the carrier frequencies of the quantum states to the desired values while keeping the quantum states unchanged [hereafter, we will focus on such kind of frequency conversion and call it quantum frequency conversion (QFC)]. For example, future quantum networks will likely require the ability to freely change between the wavelength of a particular flying quantum state (e.g., a telecommunication-wavelength optical field) and a stationary quantum state (e.g., a trapped atom or ion). Recently, a qubit transfer between photons of different wavelengths through the process of nonlinear upconversion was reported [1,2], and a polarization-independent frequency upconversion was also demonstrated [3]. Besides qubit, quantum continuous variables (CV) have emerged as a new tool for developing novel quantum communication and information-processing protocols. This extension is stimulated by the high efficiency in preparing and manipulating the quantum states in the CV regime as well as the unconditionalness. Many quantum CV protocols have been proposed and experimentally realized (for a review see [4]). Huang *et al.* have demonstrated QFC with pulsed twin beams of light [5]. A scheme of QFC of cw CV entangled states was also proposed [6]. It should be noted that QFC can also be done by quantum teleportation with a CV frequency nondegenerate Einstein–Podolsky–Rosen state in principle [7]. Unfortunately, sufficiently strong entanglement is necessary to obtain a high fidelity, and furthermore, sophisticated Bell measurements and displacement operations are needed. In this paper, QFC of cw CV quantum states via second-order nonlinear optical processes is theoretically analyzed. Particularly, the influence of vacuum noises and excess amplitude and phase noises of the strong pump field was considered.The paper is structured as follows. In Section 2, we analyze the QFC based on sum-frequency generation (SFG); in Section 3, the QFC based on difference-frequency generation (DFG) was presented; and in Section 4, we summarize and conclude.

## 2. QFC BASED ON SFG

For a cw nonlinear parametric interaction process, a cavity is usually utilized to improve significantly the conversion efficiency [8]. Consider a SFG process as shown in Fig. 1; a ring cavity is used to enhanced greatly the circulating pump power (angular frequency $\omega_2$), and it is single pass for the signal field (angular frequency $\omega_1$) and the upconverted field (angular frequency $\omega_0$) [3]. Such cavity design ensures high nonlinear conversion efficiency as well as the broadband QFC. To analyze the QFC, we adopt the semiclassical approach [9]. Assume the round-trip losses (linear and nonlinear losses) for the strong pump field are small, and therefore the pump field is spatially invariant. The evolution equations for this system can be given by [9,10]

$$\frac{\mathrm{d}\alpha_2(t)}{\mathrm{d}t} = -\gamma\alpha_2(t) + \frac{ig}{\tau}\int_0^L \alpha_0(z,t)\alpha_1^*(z,t)\mathrm{d}z + \sqrt{2\gamma_2/\tau}\alpha_2^{in} + \sqrt{2\mu_2/\tau}b_2, \tag{1}$$

$$\frac{\partial\alpha_1(z,t)}{\partial z} = ig\alpha_0(z,t)\alpha_2^*(t), \tag{2}$$

$$\frac{\partial\alpha_0(z,t)}{\partial z} = ig\alpha_1(z,t)\alpha_2(t), \tag{3}$$

where $\alpha_j(z,t)$ stand for the slow-varying field amplitudes ($j=0$ for upconverted, $j=1$ for the signal, $j=2$ for the

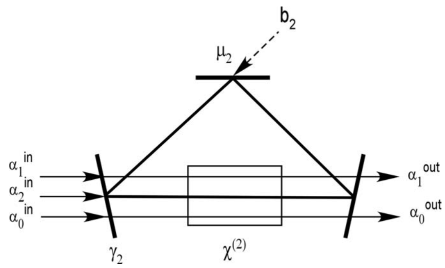


Fig. 1. Broadband QFC of cw CV quantum states based on SFG (DFG).

pump), and they satisfy the relation $\alpha_j(z,t) = \sqrt{\varepsilon_0 c n_j/(2\hbar\omega_j)}E_j$ ($E_j$ are actual electric fields). So the variable $|\alpha_j(z,t)|^2$ gives the photon flux. $\alpha_2^{in}$ is the input pump field amplitude, and $b_2$ is the vacuum field due to the linear losses. $g=\chi^{(2)}[2\hbar\omega_0\omega_1\omega_2/(\varepsilon_0 c^3 n_1 n_2 n_3)]^{1/2}$ is the nonlinear coupling coefficient, where $n_j$ is the refractive index at frequency $\omega_j$, $c$ is the speed of light in vacuum, $\chi^{(2)}$ is the second-order effective nonlinear susceptibility, and $\varepsilon_0$ is the dielectric constant of vacuum. $\tau$ is the cavity round-trip time of the pump field; $\gamma_2$, $\mu_2$ are the input coupling and the intracavity linear loss rates for the pump field, respectively; and $\gamma=\gamma_2+\mu_2$ is the total loss rate. $L$ is the length of the nonlinear crystal.

By combining Eqs. (2) and (3), The expressions for the $\alpha_1(z,t)$ and $\alpha_0(z,t)$ can be derived:

$$\alpha_1(z,t) = \alpha_1^{in}\cos(g|\alpha_2(t)|z) + i\alpha_0^{in}\frac{|\alpha_2(t)|}{\alpha_2(t)}\sin(g|\alpha_2(t)|z), \quad (4)$$

$$\alpha_0(z,t) = i\alpha_1^{in}\frac{\alpha_2(t)}{|\alpha_2(t)|}\sin(g|\alpha_2(t)|z) + \alpha_0^{in}\cos(g|\alpha_2(t)|z), \quad (5)$$

where $\alpha_1^{in}=\alpha_1(0,t)$, $\alpha_0^{in}=\alpha_0(0,t)$ are the input signal and upconverted fields amplitude at the input surface of the nonlinear crystal, respectively. We assume $\alpha_0^{in}$ is in the vacuum state. When the pump field is treated as a perfectly coherent monochromatic field with a stabilized intensity and the complete nonlinear conversion has occurred at the output surface of the nonlinear crystal, i.e., $g|\alpha_2(t)|L=\pi/2$, Eq. (5) can be rewritten as

$$\alpha_0(L,t) = \exp(i\phi)\alpha_1^{in}, \quad (6)$$

where $\exp(i\phi)=i\alpha_2(t)/|\alpha_2(t)|$. Except for an unimportant absolute phase, the upconverted field will be in the same quantum state as the input signal field. Thus, perfect QFC of a cw CV quantum state is realized. Also, it is a broadband QFC, which is not limited by the cavity linewidth of the pump field. This is only an ideal situation, and in practice there are classical and quantum noises in the pump field [11,12]. Such noises will inevitably degrade the fidelity of QFC. Hereafter, starting with Eqs. (1)–(5), we will investigate the QFC of Gaussian states (of course, QFC of other quantum states can also be investigated based on these five equations). In particular we assume the input signal field is an unknown coherent state $|\alpha\rangle$ and investigate the effects of pump field noises by evaluating the fidelity [13]

$$F = \langle\alpha|\rho_{out}|\alpha\rangle = \frac{2}{[(2+N_X)(2+N_Y)]^{1/2}}, \quad (7)$$

with

$$N_X = N_X^{out} - N_X^{in}, \quad N_Y = N_Y^{out} - N_Y^{in}, \quad (8)$$

where $\rho_{out}$ is the output state, $N_j^{out}$, $N_j^{in}$ ($j=X,Y$) are quadrature noises of output and input states, respectively.

One can get the expressions of pump field $\alpha_2(t)$ by substituting Eqs. (4) and (5) into Eq. (1). For simplicity, we assume the intensity of signal field is much weaker than that of the pump field and that they satisfy the relation $|\alpha_1|^2/|\alpha_2|^2 \ll 2\gamma\tau$, $|\alpha_1|/|\alpha_2| \ll \sqrt{2\mu_2\tau}$ (this is usually satisfied in the practical system). By using this approximation, the second term on the right side of Eq. (1) can be neglected, and we have

$$\frac{d\alpha_2(t)}{dt} = -\gamma\alpha_2(t) + \sqrt{2\gamma_2/\tau}\alpha_2^{in} + \sqrt{2\mu_2/\tau}b_2. \quad (9)$$

In frequency space, the solution for the pump field is

$$\alpha_2(\omega) = \frac{\sqrt{2\gamma_2/\tau}\alpha_2^{in}(\omega) + \sqrt{2\mu_2/\tau}b_2(\omega)}{-i\omega+\gamma}. \quad (10)$$

To determine the fluctuation of $\alpha_0(L,t)$, we make use of the technique of linearization, where the amplitudes of optical fields are expanded in terms of mean fields and small fluctuations. Assuming a complete frequency conversion and neglecting higher order terms, the fluctuation of the upconverted field can be obtained from Eq. (5):

$$\delta\alpha_0(L,t) = \exp(i\langle\theta\rangle)(i\delta\alpha_1^{in} - \delta\theta A_1^{in}), \quad (11)$$

where $A_1^{in}=\langle\alpha_1^{in}\rangle$ and $\delta\alpha_0(L,t)$ and $\delta\alpha_1^{in}$ are field fluctuations for the output upconverted and input signal with $\langle\delta\alpha_0(L,t)\rangle=0$ and $\langle\delta\alpha_1^{in}\rangle=0$. $\delta\theta$ is the phase fluctuation of pump field with $\exp(i\theta)=\alpha_2(t)/|\alpha_2(t)|$ and $\langle\delta\theta\rangle=0$. To assure the same quadrature is being compared in all input and output fields, we define the quadratures as $X_{\alpha_0}=\exp[-i(\langle\theta\rangle+\pi/2)]\alpha_0+\exp[i(\langle\theta\rangle+\pi/2)]\alpha_0^*$, $Y_{\alpha_0}=i(\exp[i(\langle\theta\rangle+\pi/2)]\alpha_0^*-\exp[-i(\langle\theta\rangle+\pi/2)]\alpha_0)$; $X_{\alpha_1}=\alpha_1+\alpha_1^*$, $Y_{\alpha_1}=i(\alpha_1^*-\alpha_1)$; $X_{\alpha_2}=\exp(-i\langle\theta\rangle)\alpha_2+\exp(i\langle\theta\rangle)\alpha_2^*$, $Y_{\alpha_2}=i(\exp(i\langle\theta\rangle)\alpha_2^*-\exp(-i\langle\theta\rangle)\alpha_2)$. From Eq. (11), the fluctuations of two quadratures of the upconverted field in frequency space can be given by

$$\delta X_{\alpha_0}(\omega) = \delta X_{\alpha_1}^{in}(\omega),$$

$$\delta Y_{\alpha_0}(\omega) = \delta Y_{\alpha_1}^{in}(\omega) + 2\delta\theta(\omega)A_1^{in}. \quad (12)$$

The noise spectra thus can be calculated as

$$N_{X_{\alpha_0}}(\omega) = \langle\delta X_{\alpha_0}(\omega)\delta X_{\alpha_0}^*(\omega)\rangle = N_{X_{\alpha_1}^{in}}(\omega), \quad (13)$$

$$N_{Y_{\alpha_0}}(\omega) = \langle\delta Y_{\alpha_0}(\omega)\delta Y_{\alpha_0}^*(\omega)\rangle = N_{Y_{\alpha_1}^{in}}(\omega) + 4|A_1^{in}|^2\langle\delta\theta(\omega)\delta\theta^*(\omega)\rangle. \quad (14)$$

Here, the noise spectra have been normalized ($N_X=N_Y=1$ corresponds to quantum noise limit). It is interesting that amplitude quadrature noise spectrum of upconverted field is not sensitive to the noises of both amplitude and phase quadratures of the pump field, whereas the phase quadrature noise spectrum of the upconverted field is related to only the phase noise of the pump field. From the definition of $\theta$, one can easily obtain $\delta\theta(\omega) = \delta Y_{\alpha_2}(\omega)/(2|A_2|)$, here $|A_2| = \langle \alpha_2(t) \rangle$. Thus the noise spectrum is

$$\langle \delta\theta(\omega)\delta\theta^*(\omega) \rangle = \frac{1}{4|A_2|^2} \langle \delta Y_{\alpha_2}(\omega)\delta Y^*_{\alpha_2}(\omega) \rangle. \tag{15}$$

By using Eq. (10), the phase quadrature noise spectrum of the pump field is

$$\langle \delta Y_{\alpha_2}(\omega)\delta Y^*_{\alpha_2}(\omega) \rangle = \frac{(2\gamma_2/\tau)N_{Y^{in}_{\alpha_2}}(\omega) + 2\mu_2/\tau}{\omega^2 + \gamma^2}, \tag{16}$$

where $N_{Y^{in}_{\alpha_2}}(\omega)$ is phase quadrature noise spectrum of the input pump field.

By combining Eqs. (8) and (13)–(16), we can obtain the equivalent input noises

$$N_X = 0,$$

$$N_Y = \left( \frac{|A_1^{in}|^2}{|A_2|^2} \right) \frac{(2\gamma_2/\tau)N_{Y^{in}_{\alpha_2}}(\omega) + 2\mu_2/\tau}{\omega^2 + \gamma^2}. \tag{17}$$

So the fidelity given by Eq. (7) can be rewritten as

$$F = \frac{2}{\left[ 2\left( 2 + \left( \frac{|A_1^{in}|^2}{|A_2|^2} \right) \frac{(2\gamma_2/\tau)N_{Y^{in}_{\alpha_2}}(\omega) + 2\mu_2/\tau}{\omega^2 + \gamma^2} \right) \right]^{1/2}}. \tag{18}$$

It is evident that the existence of phase fluctuations of pump field (both classical and quantum) will degrade the fidelity of the QFC, whereas the fidelity is not sensitive to the amplitude fluctuations.

Figure 2 shows the fidelity of the QFC versus the phase quadrature noise of the input pump field $N_P^{in}$ for different normalized frequencies $\omega/2\gamma = 0, 2, 10$. In Fig. 2, $N_{Y^{in}_{\alpha_2}}(\omega)$ has been assumed to be independent of frequency, and typical experimental parameters have been used with total cavity loss of $2\gamma\tau = 0.05$ and $|A_1^{in}|^2/|A_2|^2 = 0.001$. It is clear from Fig. 2 that the fidelity decreases owing to the phase quadrature noise, and the effect of phase fluctuations upon fidelity becomes weaker with increasing value of normalized frequencies. This is because the cavity in Fig. 1 acts as a spatial mode cleaner [14] as well as an enhancement cavity for high frequencies ($\omega \gg 2\gamma$), so the pump noises will be reflected by the cavity.

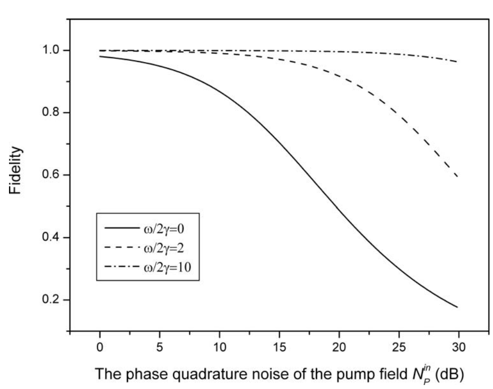


Fig. 2. The fidelity of the QFC versus the phase quadrature noise of the input pump field ($N_P^{in} = 10 \log N_{Y^{in}_{\alpha_2}}$) for $\omega/2\gamma = 0, 2, 10$. The total cavity loss is $2\gamma\tau = 0.05$ and $|A_1^{in}|^2/|A_2|^2 = 0.001$.

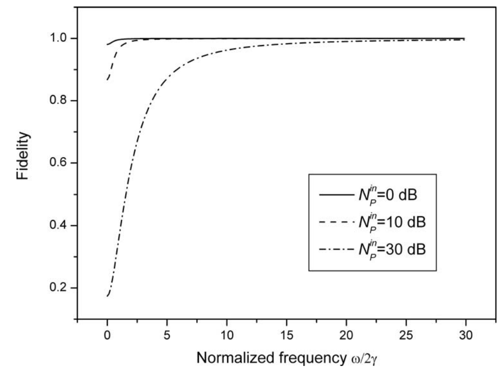


Fig. 3. Fidelity of the QFC versus the normalized frequency $\omega/2\gamma$ for different phase quadrature noise of the input pump field. The other parameters are the same as in Fig. 2.

The fidelity of the QFC versus the normalized frequency $\omega/2\gamma$ for different phase quadrature noises of the input pump field is shown in Fig. 3. The fidelity of the QFC is increased with increasing value of frequencies, such phenomena is also attributed to the noise filtering of the cavity. It should be noted that when the phase quadrature noise of the input pump field is not so large, the curve of the fidelity is fairly flat; this indicates that our QFC is essentially a broadband one, which is important for a universal QFC.

In above analysis, a perfect nonlinear conversion has been assumed. If the nonlinear conversion is not complete, vacuum field ($\alpha_0^{in}$) will enter the upconverted field, and the fidelity of the QFC will be degraded. It should be noted that there exists a tolerance for the noises of the input pump field that does not need to be an ideal coherent state. To determine the tolerance, $|A_1^{in}|^2/|A_2|^2$ is an important parameter; the smaller it is, the larger the tolerance is. For given value of parameter $|A_1^{in}|^2/|A_2|^2$, the tolerance of input pump field noise can be estimated conveniently by using Eq. (18).

## 3. QFC BASED ON DFG

For the QFC based on SFG, the carrier frequency of the frequency-converted quantum state must be larger than that of the initial quantum state, whereas it is possible to decrease the carrier frequency by exploring a DFG process. Similar to Eqs. (1)–(3), the evolution equations for QFC based on DFG can be given by

$$\frac{d\alpha_0(t)}{dt} = -\gamma\alpha_0(t) + \frac{ig}{\tau}\int_0^L \alpha_1(z,t)\alpha_2(z,t)dz + \sqrt{2\gamma_0/\tau}\alpha_0^{in} + \sqrt{2\mu_0/\tau}b_0, \tag{19}$$

$$\frac{\partial \alpha_1(z,t)}{\partial z} = ig\alpha_0(t)\alpha_2^*(z,t), \tag{20}$$

$$\frac{\partial \alpha_2(z,t)}{\partial z} = ig\alpha_0(t)\alpha_1^*(z,t), \tag{21}$$

where $\alpha_0(t)$, $\alpha_1(z,t)$, and $\alpha_2(z,t)$ stand for the slow varying complex amplitudes of pump, signal, and downconverted fields, respectively. $\alpha_0^{in}$ is the input pump field amplitude. Other parameters have the same definitions with those of SFG process.

By combining Eqs. (20) and (21), we have [15]

$$\alpha_1(z,t) = \alpha_1^{in}\cosh(g|\alpha_0(t)|z) + i\alpha_2^{in^*}\frac{\alpha_0(t)}{|\alpha_0(t)|}\sinh(g|\alpha_0(t)|z), \tag{22}$$

$$\alpha_2(z,t) = i\alpha_1^{in^*}\frac{\alpha_0(t)}{|\alpha_0(t)|}\sinh(g|\alpha_0(t)|z) + \alpha_2^{in}\cosh(g|\alpha_0(t)|z), \tag{23}$$

where $\alpha_1^{in}=\alpha_1(0,t)$, $\alpha_2^{in}=\alpha_2(0,t)$ are the input signal and downconverted fields amplitude at the input surface of the nonlinear crystal, respectively. We assume $\alpha_2^{in}$ is in the vacuum state. When the pump field is treated as a perfectly coherent monochromatic field with a stabilized intensity and the complete nonlinear conversion is occurred, Eq. (23) can be rewritten as

$$\alpha_2(L,t) = \exp(i\phi)\alpha_1^{in^*} + \sqrt{2}\alpha_2^{in}, \tag{24}$$

where $\exp(i\phi)=i\exp(i\theta)=i\alpha_0(t)/|\alpha_0(t)|$. The corresponding phase conjugation fidelity for a coherent state input is

$$F = \langle\alpha^*|\rho_{out}|\alpha^*\rangle = 1/2, \tag{25}$$

where $|\alpha^*\rangle$ is the phase conjugation state of $|\alpha\rangle$. From Eq. (25), we cannot realize a perfect phase conjugation QFC by DFG. Actually, the phase conjugation of an unknown Gaussian state cannot be realized perfectly by any physical process, and some noises will inevitably be introduced by an approximate phase conjugation operation [16]. In an ideal situation (a perfectly coherent monochromatic pump field with a stabilized intensity), the DFG can realize an optimal phase conjugation QFC (the lower bound of added noise is achieved) [16].

The classical and quantum noises in the pump field will also inevitably degrade the fidelity of phase conjugation QFC. Under the assumption of weak signal field $|\alpha_1|^2/|\alpha_0|^2 \ll 2\gamma\tau$, $|\alpha_1|/|\alpha_0| \ll \sqrt{2\mu_0\tau}$, by using Eq. (19), the quadrature noise spectra of the pump field are

$$N_{Y_{\alpha_0}}(\omega) = \frac{(2\gamma_0/\tau)N_{Y_{\alpha_0}^{in}}(\omega) + 2\mu_0/\tau}{\omega^2+\gamma^2}, \tag{26}$$

$$N_{X_{\alpha_0}}(\omega) = \frac{(2\gamma_0/\tau)N_{X_{\alpha_0}^{in}}(\omega) + 2\mu_0/\tau}{\omega^2+\gamma^2}, \tag{27}$$

where $N_{X_{\alpha_0}^{in}}(\omega)$ and $N_{Y_{\alpha_0}^{in}}(\omega)$ are noise spectra of the input pump field.

By using the technique of linearization, the fluctuation of downconverted field can be given by

$$\delta\alpha_2(L,t) = i\exp(\langle\theta\rangle)\left(\delta\alpha_1^{in^*} + \sqrt{2}\ln(1+\sqrt{2})\frac{\delta|\alpha_0(t)|}{|A_0|}A_1^{in^*} + i\delta\theta A_1^{in^*}\right) + \sqrt{2}\alpha_2^{in}, \tag{28}$$

where $A_1^{in}=\langle\alpha_1^{in}\rangle$. The noise spectra of two quadratures can be calculated as

$$N_{X_{\alpha_2}}(\omega) = N_{X_{\alpha_1}^{in}}(\omega) + C^2\frac{|A_1^{in}|^2}{|A_0|^2}N_{X_{\alpha_0}}(\omega) + 2N_{X_{\alpha_2}^{in}}(\omega), \tag{29}$$

$$N_{Y_{\alpha_2}}(\omega) = N_{Y_{\alpha_1}^{in}}(\omega) + \frac{|A_1^{in}|^2}{|A_0|^2}N_{Y_{\alpha_0}}(\omega) + 2N_{Y_{\alpha_2}^{in}}(\omega), \tag{30}$$

where $C=\sqrt{2}\ln(1+\sqrt{2})$, $A_0=\langle\alpha_0\rangle$. By combining Eqs. (26)–(30), the phase-conjugating fidelity is

$$F = \langle\alpha^*|\rho_{out}|\alpha^*\rangle = \frac{2}{[(2+N_X')(2+N_Y')]^{1/2}}, \tag{31}$$

with

$$N_X' = C^2\frac{|A_1^{in}|^2}{|A_0|^2}\frac{(2\gamma_0/\tau)N_{X_{\alpha_0}^{in}}(\omega) + 2\mu_0/\tau}{\omega^2+\gamma^2} + 2, \tag{32}$$

$$N_Y' = \frac{|A_1^{in}|^2}{|A_0|^2}\frac{(2\gamma_0/\tau)N_{Y_{\alpha_0}^{in}}(\omega) + 2\mu_0/\tau}{\omega^2+\gamma^2} + 2. \tag{33}$$

From Eqs. (31)–(33), it is clear that the phase-conjugating fidelity is sensitive to the fluctuations of both amplitude and phase quadratures of the pump field. This is also shown in Fig. 4.

Although it is impossible to obtain a perfect QFC by using DFG, we will show that any one quadrature of the in-

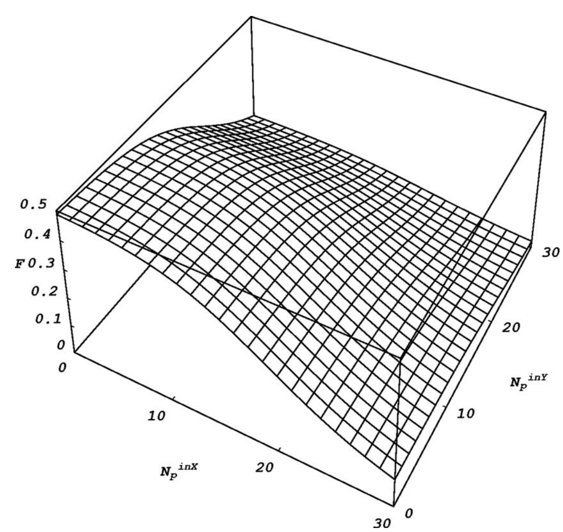


Fig. 4. The fidelity ($F$) of the QFC versus the phase and amplitude quadratures noise of the input pump field ($N_P^{inX}=10\log N_{X_{\alpha_0}^{in}}$, $N_P^{inY}=10\log N_{Y_{\alpha_0}^{in}}$) for $\omega/2\gamma=0$. The other parameters are the same as in Fig. 2.

put signal field can be transferred by constructing a feedback loop. When the complete nonlinear conversion is occurred, Eqs. (22) and (23) can be rewritten as [we assume $i\alpha_0(t)/|\alpha_0(t)|=1$]

$$\alpha_1(L,t) = \sqrt{2}\alpha_1^{in} + \alpha_2^{in^*}, \tag{34}$$

$$\alpha_2(L,t) = \alpha_1^{in^*} + \sqrt{2}\alpha_2^{in}. \tag{35}$$

The corresponding equations in the quadratures (defined as $X^{(\varphi)}=\alpha \exp(i\varphi)+\alpha^* \exp(-i\varphi)$) are then given by

$$X_{\alpha_1}^{(\varphi)} = \sqrt{2}X_{\alpha_1}^{in(\varphi)} + X_{\alpha_2}^{in(-\varphi)}, \tag{36}$$

$$X_{\alpha_2}^{(\varphi)} = X_{\alpha_1}^{in(-\varphi)} + \sqrt{2}X_{\alpha_2}^{in(\varphi)}. \tag{37}$$

It is useful to write Eq. (37) as

$$X_{\alpha_2}^{(\varphi)} = -X_{\alpha_1}^{in(-\varphi)} + \sqrt{2}X_{\alpha_1}^{(-\varphi)}. \tag{38}$$

First, one quadrature of the output signal field $X_{\alpha_1}^{(-\varphi)}$ is detected; the measured photocurrent is $i=X_{\alpha_1}^{(-\varphi)}$. Then it is used to modulate the downconverted field $X_{\alpha_2}^{(\varphi)}$ by electro-optic modulators, and the modulated field $X_{\alpha_2}^{(\varphi)M}$ can be written as

$$X_{\alpha_2}^{(\varphi)M} = -X_{\alpha_1}^{in(-\varphi)} + \sqrt{2}X_{\alpha_1}^{(-\varphi)} + gi, \tag{39}$$

where $g$ describes a gain for the transformation from classical photon current to complex field amplitude [7]. For $g=-\sqrt{2}$, $X_{\alpha_1}^{(-\varphi)}$ is eliminated in Eq. (39) and we have

$$X_{\alpha_2}^{(\varphi)M} = X_{\alpha_1}^{in(\pi-\varphi)}. \tag{40}$$

From Eq. (40), it is evident that quadrature $X_{\alpha_1}^{in(-\varphi)}$ can be transferred by constructing a feedback loop, except for a rotation of quadrature direction.

## 4. CONCLUSION

The QFC of cw CV quantum states via second-order nonlinear optical processes is theoretically analyzed in this paper. It is shown that wideband frequency conversion of the cw CV quantum state is available by SFG and DFG. Although one cannot obtain a perfect QFC by using DFG, it is feasible that any one (and only one) quadrature of the quantum state can be perfectly transferred by constructing a feedback loop. This feature has some interesting applications; for instance, it can be applied to any situation where the useful quantum information is carried by only one quadrature of the optical field. When the classical and quantum fluctuations are considered and assuming a coherent state input, it is shown that only the phase noise of the pump field will degrade the fidelity of the QFC for SFG, whereas the fidelity of QFC by using DFG is sensitive to both the amplitude and the phase noises of the pump field. Even if there are large amplitude and phase noises in the pump field, the QFC can be robust against them, given that the input signal field is fairly weak compared with the pump field. It is worth noting that the optimal phase conjugation frequency conversion by using DFG is no better than what can be achieved classically, for instance, by simultaneously measuring the two quadratures of a coherent state at the carrier frequency of $\omega_1$ and then preparing a coherent state whose phase quadrature has a flipped sign at the carrier frequency of $\omega_2$. However, it is generally difficult to prepare a quantum state even if we have all the information about it.

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## REFERENCES

1. S. Tanzilli, W. Tittel, M. Halder, O. Alibart, P. Baldi, N. Gisin, and H. Zbinden, "A photonic quantum information interface," Nature **437**, 116–120 (2005).
2. A. P. VanDevender and P. G. Kwiat, "Quantum transduction via frequency upconversion," J. Opt. Soc. Am. B **24**, 295–299 (2007).
3. M. A. Albota, Franco N. C. Wong, and J. H. Shapiro, "Polarization-independent frequency conversion for quantum optical communication," J. Opt. Soc. Am. B **23**, 918–924 (2006).
4. S. L. Braunstein and P. V. Loock, "Quantum information with continuous variables," Rev. Mod. Phys. **77**, 513–577 (2005).
5. J. M. Huang and P. Kumar, "Observation of quantum frequency conversion," Phys. Rev. Lett. **68**, 2153–2156 (1992).
6. A. H. Tan, X. J. Jia, and C. D. Xie, "Frequency conversion of an entangled state," Phys. Rev. A **73**, 033817 (2006).
7. A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, "Unconditional quantum teleportation," Science **282**, 706–709 (1998).
8. A. Ashkin, G. D. Boyd, and J. M. Dziedzic, "Resonant optical second harmonic generation and mixing," IEEE J. Quantum Electron. **QE-2**, 109–124 (1966).
9. S. Reynaud and A. Heidmann, "A semiclassical linear input output transformation for quantum fluctuations," Opt. Commun. **71**, 209–214 (1989).
10. R. L. Byer, "Optical parametric oscillator," in *Treatise in Quantum Electronics*, H. Rabin and C. L. Tang, eds. (Academic, 1975), pp. 587–702.
11. S. Höfer, A. Liem, J. Limpert, H. Zellmer, A. Tünnermann, S. Unger, S. Jetschke, H.-R. Müller, and I. Freitag, "Single-frequency master-oscillator fiber power amplifier system emitting 20 W of power," Opt. Lett. **26**, 1326–1328 (2001).
12. M. Dubois, K. C. Burr, and T. E. Drake, "Laser phase noise reduction for industrial interferometric applications," Appl. Opt. **43**, 4399–4407 (2004).
13. P. Grangier and F. Grosshans, "Quantum teleportation criteria for continuous variables," e-print quant-ph/0009079.
14. B. Willke, N. Uehara, E. K. Gustafson, R. L. Byer, P. J. King, S. U. Seel, and R. L. Savage, Jr., "Spatial and temporal filtering of a 10-W Nd:YAG laser with a Fabry–Perot ring-cavity premode cleaner," Opt. Lett. **23**, 1704–1706 (1998).
15. C. M. Caves, "Quantum limits on noise in linear amplifiers," Phys. Rev. D **26**, 1817–1839 (1982).
16. N. J. Cerf and S. Iblisdir, "Phase conjugation of continuous quantum variables," Phys. Rev. A **64**, 032307 (2001).